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Exact spin factor for a Dirac particle in a plane wave field: coordinate gauge

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Abstract

An exact and analytic Green function for a Dirac particle in interaction with an electromagnetic plane wave field, expressed in the coordinate gauge, is given by using the path integral formalism. The reparametrization invariant and supergauge invariant action describing the relativistic spinning particle is used. The problem is simplified by using two identities which relate the quantum problem directly to its classical analogue. The role of these identities is to separate the free propagation (bosonic and fermionic) from the interaction term. Obviously, the bosonic classical evolution is corrected by the spin fluctuation. From the exact expression of the Green function, the Polyakov spin factor is deduced in an explicit way.

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1. Introduction

Up to now, the path integral formulation for spin has been considered a very important but difficult task in theoretical physics. The difficulty lies in the fact that, on the one hand, the spin is a physical entity with a purely quantum behaviour, i.e. it takes exclusively discrete values. On the other hand, the path integral formulation needs some classical concepts such as trajectories which are exclusively continuous. This strange feature has intrigued many researchers and has thus attracted particular attention. Consequently, there have been many attempts to elucidate this problem. These attempts can be essentially classified into two categories of models: the commutative model and the anticommutative one. The first category of models which can be said to be bosonic uses commuting variables and this can be traced back to previous models such as the Feynman model where the one-dimensional free Dirac electron, based on the Poisson process [1], Schulman model gives a path representation, where the spin dynamics is described via a top model [2], which has been extended to the relativistic case [3]; and finally the Barut attempt in which the spin is described via internal evolution by means of degrees of freedom similar to the complex spinors [4]. The latter model is particularly interesting because it is directly related to the Dirac equation and it contains the velocity oscillations known as the

Schrödinger zitterbewgung. The second category is said to be fermionic because it is based on the anticommuting (Grassmannian) variables. This is due to Berezin and Marinov [5] who, using path integrals, have presented the Dirac propagator, in terms of Grassmanian variables, in the form of exp(i action). This model is in fact a renewal of the Fradkin model which was the first to present an action for a spinning particle based on the Grassmann variables. Without pretention, we can say that this was the first successful and audacious attempt for the correct description of the spinning point particle. Noting that the action of this model has an interesting feature owing to its gauge invariance, particularly in its reparametrization invariance and local supersymmetric form. In the last decade, Fradkin and Gitman returned to this model by starting from expressions for the propagators of the corresponding quantum field theories and then establishing a rigorous formulation of this path integral representation. This can be considered as a natural and a general method [6]. Consequently, they succeeded in postulating the following expression for the causal Green function of the spinning particle in interaction with an electromagnetic field:

$$\begin{aligned}\tilde{S}^c = \exp\left(\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right) \int_0^\infty \mathrm{d}e_0 \int \mathrm{d}\chi_0 \int \mathcal{D}x\mathcal{D}e\mathcal{D}p_e\mathcal{D}\chi\mathcal{D}p_\chi\mathfrak{D}\psi M(e) \\ \times \exp\left\{\mathrm{i}\int_0^1 \left[-\frac{\dot{x}^2}{2e} - \frac{e}{2}m^2 - g\dot{x}A(x) + \mathrm{i}egF_{\mu\nu}(x)\psi^\mu\psi^\nu\right.\right. \\ \left.\left. +\,\mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - m\psi^5\right)\chi - \mathrm{i}\psi_n\dot{\psi}^n + p_\chi\dot{\chi} + p_e\dot{e}\right]\mathrm{d}\tau + \psi_n(1)\psi^n(0)\right\}\Bigg|_{\theta=0} \end{aligned} \tag{1}$$

with x, e and p_e denoting bosonic (real) variables, θ, χ, p_χ and ψ fermionic (odd Grassmannian) variables, with the following boundary conditions:

$$x(0) = x_a \qquad x(1) = x_b \qquad e(0) = e_0 \qquad \chi(0) = \chi_0 \qquad \psi(1) + \psi(0) = \theta \tag{2}$$

and

$$M(e) = \int \mathcal{D}p\, \exp\left\{\frac{1}{2}\int_0^1 ep^2\,\mathrm{d}\tau\right\} \tag{3}$$

and

$$\mathfrak{D}\psi = \mathcal{D}\psi\left[\int_{\psi(1)+\psi(0)=\theta} \mathcal{D}\psi\, \exp\left\{\int_0^1 \psi_n\dot{\psi}^n\,\mathrm{d}\tau\right\}\right]^{-1}. \tag{4}$$

In these previous expressions and in what follows the product of two quadrivectors AB stands for $AB = A_\mu B^\mu$.

Separating the gauge-fixing term $S_{GF} = \int_0^1 [p_\chi\dot{\chi} + p_e\dot{e}]\mathrm{d}\tau$ and the boundary term $\psi_n(1)\psi^n(0)$, we get the gauge invariant action

$$S = \int_0^1 \left[-\frac{\dot{x}^2}{2e} - \frac{e}{2}m^2 - g\dot{x}A(x) + \mathrm{i}egF_{\mu\nu}(x)\psi^\mu\psi^\nu + \mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - m\psi^5\right)\chi - \mathrm{i}\psi_n\dot{\psi}^n\right]\mathrm{d}\tau \tag{5}$$

from which we easily deduce the Lagrangian classical equations of motion

$$\frac{1}{e^2}\left(\frac{\dot{x}^2}{2} - \mathrm{i}\dot{x}_\alpha\psi^\alpha\chi\right) + \mathrm{i}gF_{\mu\nu}(x)\psi^\mu\psi^\nu - \frac{m^2}{2} = 0 \tag{6}$$

$$\mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - m\psi^5\right)\chi = 0 \tag{7}$$

$$2\mathrm{i}\dot{\psi}_\alpha + 2\mathrm{i}geF_{\alpha\nu}(x)\psi^\nu - \mathrm{i}\frac{\dot{x}_\alpha}{e}\chi = 0 \tag{8}$$

$$-2\mathrm{i}\dot{\psi}^5 + \mathrm{i}m\chi = 0 \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\dot{x}_\alpha}{e}\right) + g\dot{x}^\mu F_{\mu\alpha}(x) + \mathrm{i}eg\frac{\partial}{\partial x^\alpha}F_{\mu\nu}(x)\psi^\mu\psi^\nu = 0. \tag{10}$$

Our purpose in this paper is to consider the case of a Dirac particle interacting with the plane wave field which is expressed in the coordinate gauge [7–9] by using this model. This problem has already been investigated via a bosonic path integral framework where the spin calculations have been omitted [10]; to put it another way, the spin calculus has not been carried out explicitly when using path integral techniques but has been incorporated by extending the spin-zero technique. For this reason, we are reconsidering the same problem by trying to perform correctly these spin calculations by working out all the integrations using suitable identities which relate the quantum problem to its classical analogue. In fact, these identities represent the projection of the equations (8) and (10) on the plane wave propagation vector. One identity is related to the plane wave variable which has the objective of separating the interaction from the free propagation of the bosonic part. The other identity is related to the spin variable which is introduced with the aim of separating the interaction from the free-spin propagation. Besides, they have another role which consists of making apparent the predominance of the classical evolution. Consequently, the evaluation of the causal Green function will be straightforward and the result will be very simple. Furthermore, we also wish to extract the exact structure of the Polyakov spin factor [11] relative to this problem. This latter spin factor has been, for example, exactly carried out in the constant electromagnetic field case [12], in the plane wave case [13–16], and the combination of the two configurations case [17].

To end this section, let us expose the configuration of the plane wave field. Thus, the quadripotential of the plane wave is chosen in the coordinate gauge, that is to say

$$(x - x_0)^\mu A_\mu(x) = 0 \tag{11}$$

where x_0 is an arbitrary reference point.

The advantage of this gauge choice is that having the electromagnetic tensor $F_{\mu\nu}$, the 4-vector potential A_μ is determined in a unique manner following the inversion formula

$$A_\mu(x) = \int_0^1 \mathrm{d}\alpha\, \alpha(x - x_0)^\nu F_{\mu\nu}(\alpha x) \tag{12}$$

with the electromagnetic plane wave tensor having the form

$$F_{\mu\nu}(x) = f_{\mu\nu}F(\zeta) \tag{13}$$

where $\zeta = \eta x$, $F(\zeta)$ is an arbitrary function of ζ and $f_{\mu\nu}$ is a constant antisymmetric tensor verifying with η_μ the following useful properties:

$$\begin{array}{lll} \eta^\mu\eta_\mu = 0 & \eta_\mu f^{\mu\nu} = 0 & \eta_\mu f^{\mu\nu *} = 0 \\ f^*_{\mu\lambda}f^\lambda_\nu = 0 & f^*_{\mu\lambda}f^{\lambda *}_\nu = f_{\mu\lambda}f^\lambda_\nu = \eta_\mu\eta_\nu & \end{array} \tag{14}$$

f^* is the dual tensor of f.

In this gauge, the 4-potential A_μ will obviously take the following form:

$$A_\mu(x) = f_{\mu\nu}(x - x_0)^\nu K(\zeta, \zeta_0) \tag{15}$$

where $K(\zeta, \zeta_0)$ satisfies the equation

$$2K + (\zeta - \zeta_0)\frac{\mathrm{d}K}{\mathrm{d}\zeta} = -F(\zeta) \tag{16}$$

which has the following solution:

$$K(\zeta,\zeta_0) = -\frac{A(\zeta)}{(\zeta-\zeta_0)} + \frac{1}{(\zeta-\zeta_0)^2}\int_{\zeta_0}^{\zeta} \mathrm{d}\xi' A(\xi') \tag{17}$$

with $\frac{\mathrm{d}A(\zeta)}{\mathrm{d}\zeta} = F(\zeta)$.

In the following section we are going to use the path integral representation for the Dirac equation given by expression (1) to evaluate the exact and analytic Green function for a Dirac particle interacting with the plane wave field and having the previous properties.

2. Path integral calculation

Before beginning our calculation, it is suitable at first to fix in equation (1) the gauge over bosonic and Grassmanian proper times by performing functional integrations over π and ν which give, respectively, the delta functional $\delta(\dot{e})$ and $\delta(\dot{\chi})$ implying that

$$\begin{aligned} \dot{e} &= 0 \qquad e_0 = e_1 = \cdots = e_N = e \\ \dot{\chi} &= 0 \qquad \chi_0 = \chi_1 = \cdots = \chi_N = \chi. \end{aligned} \tag{18}$$

Before performing the calculations in the phase space, we go over the Hamiltonian form by completing the square in $\dot{x}$ and linearizing the arising quadratic term. The result will be

$$\begin{aligned} \tilde{S}^c(x_b,x_a) = {} & \exp\left[\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right]\int_0^\infty \mathrm{d}e \int \mathrm{d}\chi \int \mathcal{D}x\mathcal{D}p\mathfrak{D}\psi \exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau\Big[p\dot{x} - \mathrm{i}\psi_n\dot{\psi}^n \right. \\ & + \frac{e}{2}(p^2-m^2) + egpA(x) + \frac{e}{2}g^2A^2(x) - \mathrm{i}[(p+gA(x))\psi + m\psi^5]\chi \\ & \left. + \mathrm{i}egF_{\mu\nu}(x)\psi^\mu\psi^\nu\Big] + \psi_n(1)\psi^n(0)\right\}\Bigg|_{\theta=0}. \end{aligned} \tag{19}$$

Now, in order to separate the bosonic interaction terms from the free bosonic evolution we introduce a new variable ζ which considers the plane wave variable ηx as independent from the quadriposition x via the following easily proved identity:

$$\int \mathrm{d}\zeta_a\,\mathrm{d}\zeta_b\delta(\zeta_a - \eta x_a)\int \mathcal{D}\zeta\mathcal{D}p_\zeta \exp\left[\mathrm{i}\int_0^1 \mathrm{d}\tau\, p_\zeta(\dot{\zeta} - \eta\dot{x})\right] = 1. \tag{20}$$

By inserting this in equation (19), then shifting the momentum from p to $p + p_\zeta\eta$ by taking into account the equations $\eta^2 = 0$ and $\eta A = 0$, the Green function will take the following form:

$$\begin{aligned} \tilde{S}^c(x_b,x_a) = {} & \exp\left[\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right]\int_0^\infty \mathrm{d}e\int \mathrm{d}\chi\int \mathrm{d}\zeta_a\,\mathrm{d}\zeta_b\,\delta(\zeta_a - \eta x_a)\int \mathcal{D}\zeta\mathcal{D}p_\zeta\mathfrak{D}\psi \\ & \times \exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau\left[p_\zeta\dot{\zeta} - \mathrm{i}\psi_n\dot{\psi}^n - \frac{e}{2}g^2(\zeta-\zeta_0)^2K^2(\zeta,\zeta_0)\right.\right. \\ & \left.\left. - \mathrm{i}\left(p_\zeta(\eta\psi) + m\psi^5\right)\chi + \mathrm{i}egF_{\mu\nu}(\zeta)\psi^\mu\psi^\nu\right] + \psi_n(1)\psi^n(0)\right\}\tilde{S}_0(x_b,x_a)\Bigg|_{\theta=0} \end{aligned} \tag{21}$$

where $\tilde{S}_0(x_b,x_a)$ is the free propagation function defined as

$$\begin{aligned} \tilde{S}_0(x_b,x_a) = {} & \int \mathcal{D}q\mathcal{D}p\exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau\left[p\dot{q} + g(ep + \mathrm{i}\chi\psi)fqK(\zeta,\zeta_a)\right.\right. \\ & \left.\left. + ep_\zeta(p\eta) + \frac{e}{2}(p^2-m^2) - \mathrm{i}(p\psi)\chi\right]\right\}. \end{aligned} \tag{22}$$

Noting that we have replaced, in this latter expression, the 4-potential $A_\mu(x)$ with its explicit form equation (15) and have assumed that $q = x - x_0$.

At this level, the q functional integration can easily be performed to give a delta functional and as a result the free propagator is therefore reduced to

$$\tilde{S}_0(x_b, x_a) = \int \mathcal{D}p\delta[\dot{p} - g(ep + \mathrm{i}\chi\psi) f K(\zeta, \zeta_a)] \exp\left\{\mathrm{i}p(1)q(1) - \mathrm{i}p(0)q(0) + \mathrm{i}\int_0^1 \mathrm{d}\tau\left[\frac{e}{2}(p^2 - m^2) + ep_\zeta(p\eta) - \mathrm{i}(p\psi)\chi\right]\right\}. \tag{23}$$

The presence of the delta functional in this equation has the effect of selecting a differential equation satisfied by p and which is in fact nothing but the classical equation of motion. This differential equation can be integrated to produce the free propagation function in the following form:

$$\tilde{S}_0(x_b, x_a) = \int \frac{\mathrm{d}^4 p_k}{(2\pi)^4}\left[\det\left(\frac{\mathrm{d}p(1)}{\mathrm{d}p_k}\frac{\mathrm{d}p(0)}{\mathrm{d}p_k}\right)\right]^{1/2} \exp\left\{\mathrm{i}p(1)(x_b - x_a) + \mathrm{i}\int_0^1 \mathrm{d}\tau\left[\frac{e}{2}(p^2 - m^2) + ep_\zeta(p\eta) - \mathrm{i}(p\psi)\chi\right]\right\} \tag{24}$$

with $x_0 = x_a$.

Obviously, the momentum $p(\tau)$ satisfies the equation of motion in the delta functional argument presented in equation (23),

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} = egK(\zeta, \zeta_a)(pf) + \mathrm{i}\chi gK(\zeta, \zeta_a)(\psi f). \tag{25}$$

This can be solved with the iteration method yielding the following result:

$$\begin{aligned} p(\tau) = p(1) - eg(p(1)f)\int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a) + \frac{(eg)^2}{2}(p(1)\eta)\eta\left(\int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a)\right)^2 \\ + \mathrm{i}e\chi g^2\eta\int_\tau^1 \mathrm{d}s\int_s^1 \mathrm{d}s' K(\zeta(s), \zeta_a)(\eta\psi(s'))K(\zeta(s'), \zeta_a) \\ - \mathrm{i}\chi g\int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a)(\psi(s)f) \end{aligned} \tag{26}$$

where we have made use of the relations given by (14) and paid attention to the fact that

$$\eta p(\tau) = \eta p(1). \tag{27}$$

This equation is nothing but the classical equation of motion. In fact, as $p = -\frac{\dot{x}}{e} - gA(x) + \mathrm{i}\frac{1}{e}\psi\chi$ the classical equations (8) and (10) projected on the η direction give $(p\eta) = \text{constant}$.

Next, it is suitable to choose in equation (24) $p_k = p(1)$ as an initial condition for our problem. Thus, the determinant can be written as

$$\left[\det\left(\frac{\mathrm{d}p(1)}{\mathrm{d}p_k}\frac{\mathrm{d}p(0)}{\mathrm{d}p_k}\right)\right] = \det\left(\frac{\mathrm{d}p(0)}{\mathrm{d}p(1)}\right) = \det(1 + Q) \tag{28}$$

where the matrix Q is defined by

$$Q_\mu{}^\nu(\zeta) = -egf_\mu{}^\nu\int_0^1 \mathrm{d}s\, K(\zeta(s), \zeta_a) + \frac{(eg)^2}{2}\eta_\mu\eta^\nu\left(\int_0^1 \mathrm{d}s\, K(\zeta(s), \zeta_a)\right)^2. \tag{29}$$

In order to compute this determinant, let us first write

$$\det(1 + Q) = \exp \operatorname{Tr} \ln(1 + Q). \tag{30}$$

Next, we expand it in powers of Q as follows:

$$\ln(1+Q) = Q - Q^2/2 \qquad (Q^n = 0 \quad \text{for } n \geqslant 3 \quad \text{because of} \quad \eta^2 = 0 = \eta f). \tag{31}$$

Finally, since f is an antisymmetric tensor, we get

$$\mathrm{Tr}\,\ln(1+Q) = 0 \tag{32}$$

As a result, we get the determinant which is equal to unity, namely

$$\det\left(\frac{\mathrm{d}p(0)}{\mathrm{d}p(1)}\right) = 1. \tag{33}$$

Incidentally, this result is not fortuitous because this determinant contains, in principle, the quantum fluctuations [10] which are absent in our case.

Now, substituting these results in equation (21), the path integral is reduced to

$$\begin{aligned}
\tilde{S}^c(x_b, x_a) = {} & \exp\left[\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right] \int_0^\infty \mathrm{d}e \int \mathrm{d}\chi \int \mathrm{d}\zeta_a\, \mathrm{d}\zeta_b \delta(\zeta_a - \eta x_a) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} D(e, p) \\
& \times \int \mathcal{D}\zeta \mathcal{D}p_\zeta \mathfrak{D}\psi \exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau \left[p_\zeta(\dot{\zeta} + (ep\eta)) - \mathrm{i}\psi_n\dot{\psi}^n\right.\right. \\
& - \frac{e}{2} g^2(\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a) - \mathrm{i}\left[p_\zeta(\eta\psi) + m\psi^5\right]\chi + \mathrm{i}egF(\zeta)\psi^\mu f_{\mu\nu}\psi^\nu \\
& - \mathrm{i}(p\psi)\chi - \mathrm{i}(eg)\chi(pf\psi(\tau)) \int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a) + \mathrm{i}\frac{(eg)^2}{2}\chi(p\eta)(\eta\psi) \\
& \times \left(\int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a)\right)^2 + \mathrm{i}e^2 g^2 \chi(p\eta) \int_\tau^1 \mathrm{d}s \int_s^1 \mathrm{d}s' K(\zeta(s), \zeta_a)\eta\psi(s') \\
& \left.\left.\times K(\zeta(s'), \zeta_a) - \mathrm{i}eg\chi \int_\tau^1 \mathrm{d}s\, K(\zeta(s), \zeta_a)\psi(s) fp\right] + \psi_n(1)\psi^n(0)\right\}\Bigg|_{\theta=0}
\end{aligned} \tag{34}$$

where $D(e, p)$ is the free propagation function given by

$$D(e, p) = \exp\left\{\mathrm{i}p(x_b - x_a) + \mathrm{i}\left[\frac{e}{2}(p^2 - m^2)\right]\right\}. \tag{35}$$

At this stage, as can be seen, the main difficulty lies in the parts relative to the spin propagation. Here again, in order to be able to extract the free-propagation function of spin, we insert another identity which considers $\eta\psi$ as independent of ψ. As we will see later, this identity gives rise to two classical equations of motion which are, respectively, the projection of the exterior motion presented by the x coordinate and the internal (spinorial) one described by ψ along the plane wave propagation vector η. In fact, these classical equations will be decisive and very useful in computing the propagator.

Indeed, we introduce a new Grassmannian variable λ as an independent variable from ψ via the following identity:

$$\int \mathrm{d}\lambda_a\, \mathrm{d}\lambda_b\, \delta(\lambda_a - \eta\psi_a) \int \mathcal{D}\lambda \mathcal{D}p_\lambda \exp\left[\mathrm{i}\int_0^1 \mathrm{d}\tau\, p_\lambda(\dot{\lambda} - \eta\dot{\psi})\right] = 1 \tag{36}$$

where $\{\lambda, p_\lambda\}$ are Grassmannian odd variables. The causal Green function will then take the following form:

$$\tilde{S}^c(x_b, x_a) = \exp\left[\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right]\int_0^\infty \mathrm{d}e \int d\chi \int \mathrm{d}\zeta_a\,\mathrm{d}\zeta_b\,\mathrm{d}\lambda_a\,\mathrm{d}\lambda_b\,\delta(\zeta_a - \eta x_a)\int \frac{\mathrm{d}^4 p}{(2\pi)^4} D(e, p)$$
$$\times \int \mathcal{D}\zeta\mathcal{D}p_\zeta\mathcal{D}\lambda\mathcal{D}p_\lambda\mathfrak{D}\psi\,\delta(\lambda_a - \eta\psi_a)\exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau\left[p_\zeta(\dot{\zeta} + (ep\eta) - \mathrm{i}\lambda\chi)\right.\right.$$
$$+ p_\lambda(\dot{\lambda} - \eta\dot{\psi}) - \mathrm{i}\psi_n\dot{\psi}^n - \frac{e}{2}g^2(\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a) - \mathrm{i}[p\psi + m\psi^5]\chi$$
$$+ \mathrm{i}egF(\zeta)\psi^\mu f_{\mu\nu}\psi^\nu - \mathrm{i}(eg)\chi(pf\psi(\tau))\int_\tau^1 \mathrm{d}s\,K(\zeta(s), \zeta_a) + \mathrm{i}\frac{(eg)^2}{2}\chi(p\eta)$$
$$\times\lambda(\tau)\left(\int_\tau^1 \mathrm{d}s\,K(\zeta(s), \zeta_a)\right)^2 + \mathrm{i}e^2g^2\chi(p\eta)\int_\tau^1 \mathrm{d}s\int_s^\tau \mathrm{d}s' K(\zeta(s), \zeta_a)$$
$$\left.\times\,\lambda(s')K(\zeta(s'), \zeta_a) - \mathrm{i}eg\chi\int_\tau^1 \mathrm{d}s\,K(\zeta(s), \zeta_a)(\psi(s)\,fp)\right]$$
$$\left.\left. + \psi_n(1)\psi^n(0)\right\}\right|_{\theta=0}. \tag{37}$$

Now, it is clear that the integrations over p_ζ may give a delta functional involving the following equation:

$$\dot{\zeta} + (ep\eta) - \mathrm{i}\lambda\chi = 0 \tag{38}$$

which is exactly the classical equation of motion projected along the η direction. In fact, it is $p = -\frac{\dot{x}}{e} - gA(x) + \mathrm{i}\frac{1}{e}\psi\chi$ projected on the wave propagation vector η. It is important to note that it is extracted in a natural way here. That is to say, the classical behaviour of the particle is predominant in the propagation. Hereafter, the variable ζ will be considered as a parameter of the evolution.

In order to get the classical equation of motion satisfied by the spin variables, we have first to get rid of the boundary condition $\psi^n(1) + \psi^n(0) = 0$, which is the spin antiperiodic condition. For this, we go over the velocity space ω related to ψ via the following replacement:

$$\psi(\tau) = \frac{1}{2}\left(\int_0^1 \mathrm{d}\tau'\,\epsilon(\tau - \tau')\omega(\tau') + \theta\right) \tag{39}$$

in other words

$$\dot{\psi}(\tau) = \omega(\tau)\,. \tag{40}$$

The change given by equation (39) is to satisfy the boundary condition for any $\omega(\tau)$ eliminating therefore any restriction on the velocities. As a result, the propagation function relative to the ω^μ velocity space will have a Gaussian form and is written as

$$I(\omega^\mu) = \int \mathfrak{D}\omega^\mu \exp\left\{-\frac{1}{2}\int_0^1 \mathrm{d}\tau\int_0^1 \mathrm{d}\tau'\,\omega^\mu(\tau)F_{\mu\nu}(\tau, \tau')\omega^\nu(\tau) + \int_0^1 \mathrm{d}\tau\,J_\mu(\tau)\,\omega^\mu(\tau)\right\} \tag{41}$$

where

$$F_{\mu\nu}(\tau, \tau') = g_{\mu\nu}\epsilon(\tau - \tau') - \frac{e}{2}gf_{\mu\nu}\int_0^1 \mathrm{d}s\,F(\zeta(s))\epsilon(\tau - s)\epsilon(s - \tau') \tag{42}$$

where $g_{\mu\nu}$ denotes the metric tensor and $J_\mu(\tau)$ is an odd source given by

$$J_\mu(\tau) = -\mathrm{i}p_\lambda\eta_\mu + \frac{\mathrm{i}}{2}p_{\lambda_a}\eta_\mu - \chi(p_\mu/2)\int_0^1 \mathrm{d}\tau'\epsilon(\tau'-\tau) - \frac{eg}{2}\int_0^1 \mathrm{d}\tau'\, F(\zeta(\tau'))$$
$$\times\,\epsilon(\tau'-\tau)\theta^\nu f_{\nu\mu} + \frac{eg}{2}\chi p^\nu f_{\nu\mu}\int_0^1 \mathrm{d}\tau'\,\epsilon(\tau'-\tau)\int_{\tau'}^1 \mathrm{d}s\, K(\zeta(s),\zeta_a)$$
$$-\frac{eg}{2}\chi p^\nu f_{\nu\mu}\int_0^1 \mathrm{d}\tau'\int_{\tau'}^1 \mathrm{d}s\, K(\zeta(s),\zeta_a)\epsilon(s-\tau). \tag{43}$$

For the $\omega^5(\tau)$ velocity space, we will have to evaluate the following Gaussian integral:

$$I(\omega^5) = \int \mathfrak{D}\omega^5 \exp\left\{-\frac{1}{2}\int_0^1 \mathrm{d}\tau\int_0^1 \mathrm{d}\tau'\,\omega^5(\tau)\epsilon(\tau-\tau')\omega^5(\tau') + \int_0^1 \mathrm{d}\tau\, J_5(\tau)\omega^5(\tau)\right\} \tag{44}$$

where

$$J_5(\tau) = -\frac{m}{2}\chi\int_0^1 \mathrm{d}\tau'\,\epsilon(\tau'-\tau). \tag{45}$$

In order to extract the classical equation over the spin variables let us introduce the following shift of velocity $\omega^\mu(\tau)$:

$$\omega^\mu(\tau) \longrightarrow \omega^\mu(\tau) + \mathrm{i}\eta^\mu\int_0^1 \mathrm{d}\tau'\,\epsilon^{-1}\left(\tau-\tau'\right)p_\lambda(\tau'). \tag{46}$$

In this manner, making use of the plane wave properties $\eta^2 = 0$ and $\eta f = 0$, we obtain the following replacement:

$$I(\omega^\mu) \longrightarrow I'(\omega^\mu)\exp\left\{\mathrm{i}\int_0^1 \mathrm{d}\tau\; p_\lambda(p\eta)\frac{\chi}{2}\right\} \tag{47}$$

where

$$I'(\omega^\mu) = I(\omega^\mu)\mid_{J_\mu(\tau)\to J'_\mu(\tau)} \tag{48}$$

with $J'_\mu(\tau) = J_\mu(\tau) + \mathrm{i}p_\lambda\eta_\mu$.

Now, the integration over p_ζ and p_λ can easily and directly be performed giving the following products of the delta functional:

$$\Pi\delta(\dot\zeta + e(p\eta) - \mathrm{i}\lambda\chi) \tag{49}$$

$$\Pi\delta\left(\dot\lambda + (p\eta)\frac{\chi}{2}\right). \tag{50}$$

Consequently, these expressions impose two constraints

$$\dot\zeta + e(p\eta) - \mathrm{i}\lambda\chi = 0 \tag{51}$$

$$\dot\lambda + (p\eta)\frac{\chi}{2} = 0 \tag{52}$$

which are the classical equations of motion (8) and (10) projected along the η direction. In fact, by using the plane wave properties given by (14) , the classical equations (8) and (10) give $\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\eta\dot x}{e}\right) = 0$ and $2\mathrm{i}\eta\dot\psi - \mathrm{i}\frac{\eta\dot x}{e}\chi = 0$, and from $p\eta = -\frac{\eta\dot x}{e} + \mathrm{i}\frac{1}{e}\eta\psi\chi$ we get the result. Namely, the classical dynamics dominate the quantum evolution. Finally, the solutions of these equations which will be very useful later on are given as

$$\zeta(\tau) = \zeta_a - e(p\eta)\tau + \mathrm{i}\lambda_a\chi \tag{53}$$

$$\lambda(\tau) = \lambda_a - (p\eta)\frac{\chi}{2}\tau. \tag{54}$$

At this stage, we are ready to perform the Gaussian integrals $I(\omega^5)$ and $I(\omega^\mu)$. The result is

$$I(\omega^5) = \exp\left\{-\frac{1}{2}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau'\, J_5(\tau)\epsilon^{-1}(\tau-\tau')J_5(\tau')\right\} = 1 \tag{55}$$

$$I(\omega^\mu) = \left(\frac{\det F}{\det F\,|_{g=0}}\right)^{\frac{1}{2}} \exp\left\{-\frac{1}{2}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau' J^\mu(\tau)F^{-1}_{\mu\nu}(\tau,\tau')J^\mu(\tau')\right\} \tag{56}$$

where we have made the usual shifts

$$\omega^5(\tau) \longrightarrow \omega^5(\tau) + \int_0^1 \mathrm{d}\tau' J^5(\tau')\epsilon^{-1}(\tau'-\tau) \tag{57}$$

$$\omega^\mu(\tau) \longrightarrow \omega^\mu(\tau) + \int_0^1 \mathrm{d}\tau' J^\mu(\tau')F^{-1}_{\mu\nu}(\tau',\tau). \tag{58}$$

Now, let us evaluate the quantity $\left(\frac{\det F}{\det F|_{g=0}}\right)^{\frac{1}{2}}$ involved in equation (56). For this purpose, we first differentiate the well known formula

$$\det F = \exp(\mathrm{Tr}\,\ln F) \tag{59}$$

with respect to g and then get

$$\frac{\mathrm{d}}{\mathrm{d}g}\det F = \det F\ \mathrm{Tr}\left(F^{-1}\frac{\mathrm{d}F}{\mathrm{d}g}\right). \tag{60}$$

This can be integrated to give

$$\left(\frac{\det F}{\det F\,|_{g=0}}\right)^{\frac{1}{2}} = \exp\left[\frac{1}{2}\int_0^g \mathrm{d}g'\,\mathrm{Tr}\left(F^{-1}\frac{\mathrm{d}F}{\mathrm{d}g'}\right)\right]. \tag{61}$$

Next, knowing that the inverse matrix F^{-1} verifies the following requirement:

$$\int_0^1 \mathrm{d}\tau'' F^{\mu\lambda}(\tau,\tau'')F^{-1}_{\lambda\nu}(\tau'',\tau') = \delta^\mu_\nu\delta(\tau-\tau') \tag{62}$$

we substitute $F_{\mu\nu}$ by its expression (42) and make the following replacement:

$$\Omega^\mu_\nu(\tau,\tau') = \int_0^1 \mathrm{d}\tau''\delta^{\mu\lambda}\epsilon(\tau-\tau'')F^{-1}_{\lambda\nu}(\tau'',\tau'). \tag{63}$$

Then, equation (62) converts to an integral equation in terms of $\Omega_{\mu\nu}(\tau,\tau')$ as

$$\Omega^\mu_\nu(\tau,\tau') = \delta_{\mu\nu}\delta(\tau-\tau') + \frac{e}{2}gf_{\mu\lambda}\int_0^1 \mathrm{d}s\, F(\zeta\,(s))\epsilon\,(\tau-s)\,\Omega^\lambda{}_\nu(s,\tau'). \tag{64}$$

Thanks to the plane wave properties, the integration of this equation ends at the second iteration. Thus, the result is

$$\Omega^\mu_\nu(\tau,\tau') = \delta^\mu_\nu\delta(\tau-\tau') + \frac{e}{2}gf^\mu_\lambda F(\zeta(\tau'))\epsilon(\tau-\tau') + \left(\frac{eg}{2}\right)^2\eta^\mu\eta_\nu\int_0^1 \mathrm{d}s\, F(\zeta(s))F(\zeta(\tau'))\epsilon(\tau-s)\epsilon(s-\tau'). \tag{65}$$

One can easily check that $F^{-1}_{\mu\nu}(\tau,\tau')$ can be found by inverting equation (63):

$$F^{-1}_{\mu\nu}(\tau,\tau') = \int_0^1 \mathrm{d}\tau''\, g_{\mu\lambda}\epsilon^{-1}(\tau-\tau'')\Omega^{\lambda}_{\nu}(\tau'',\tau'). \tag{66}$$

So substituting (65), we arrive at the final expression

$$\begin{aligned} F^{-1}_{\mu\nu}(\tau,\tau') &= g_{\mu\nu}\epsilon^{-1}(\tau-\tau') + \frac{e}{2} g f_{\mu\nu} F(\zeta(\tau'))\delta(\tau-\tau') \\ &\quad + \left(\frac{eg}{2}\right)^2 \eta_\mu\eta_\nu F(\zeta(\tau))F(\zeta(\tau'))\epsilon(\tau-\tau') \end{aligned} \tag{67}$$

and hence

$$\left(\frac{\det F}{\det F\,|_{g=0}}\right)^{\frac{1}{2}} = 1. \tag{68}$$

Notice that this result is obtained with the help of the relations (14) and by taking into account the fact that f is an antisymmetric tensor.

Now, by substituting all these changes into (56), we obtain for $I(\omega^\mu)$ the following expression:

$$I(\omega^\mu) = \exp\left\{\frac{-\mathrm{i}}{4} p_{\lambda_a}(p\eta)\chi - \frac{eg}{4}\chi(\theta f p)h(1,0) + \left(\frac{eg}{2}\right)^2 \chi(\eta\theta)(p\eta)H(1,0)\right\} \tag{69}$$

where

$$h(1,0) = \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau'\, F(\zeta(\tau')\epsilon(\tau'-\tau) \tag{70}$$

and

$$\begin{aligned} H(1,0) &= \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau' \int_{\tau'}^1 \mathrm{d}\tau'' K(\zeta(\tau''),\zeta_a)\epsilon(\tau'-\tau)F(\zeta(\tau)) \\ &\quad - \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau' \int_{\tau'}^1 \mathrm{d}\tau''\, K(\zeta(\tau''),\zeta_a)\epsilon(\tau''-\tau)F(\zeta\,(\tau)) \\ &\quad - \frac{1}{2}\int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}\tau' \int_0^1 \mathrm{d}\tau'' F(\zeta(\tau))\epsilon(\tau-\tau')F(\zeta(\tau'))\epsilon(\tau'-\tau''). \end{aligned} \tag{71}$$

Using the explicit solutions of the classical equations of motion (51) and (52) and combining all the previous results, and after straightforward and long computations, the propagator will be reduced to

$$\begin{aligned} \tilde{S}(x_b,x_a) &= \exp\left(\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right) \int \mathrm{d}e \int \mathrm{d}\chi \int \frac{\mathrm{d}^4 p}{(2\pi)^4} D(e,p) \int \mathrm{d}\zeta_b\, \mathrm{d}\zeta_a\, \mathrm{d}\lambda_b\, \mathrm{d}\lambda_a\, \delta(\zeta_a - \eta x_a) \\ &\quad \times \delta\left(\lambda_b - \lambda_a + \frac{p\eta}{2}\chi\right)\delta\left(\lambda_a - \frac{\eta\theta}{2} - \frac{p\eta}{4}\chi\right)\,\delta\left(\zeta_b - \zeta_a + ep\eta - \mathrm{i}\frac{\eta\theta}{2}\chi\right) \\ &\quad \times \exp\left\{\mathrm{i}\frac{g^2}{2p\eta}\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta(\zeta-\zeta_a)^2 K^2(\zeta,\zeta_a) + \frac{1}{2}(p\theta + m\theta^5)\chi + \frac{g^2}{4(p\eta)}\frac{(\eta\theta)\chi}{(\zeta_b-\zeta_a)}\right. \\ &\quad \times \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta(\zeta-\zeta_a)^2 K^2(\zeta,\zeta_a) + \frac{g}{4(p\eta)}(A(\zeta_b) - A(\zeta_a))(\theta f\theta) \\ &\quad - \frac{\mathrm{i}(\eta\theta)\chi}{8(p\eta)(\zeta_b-\zeta_a)}(A(\zeta_b) - A(\zeta_a))(\theta f\theta) - \frac{g}{4(p\eta)}(\theta f p)\frac{\tilde{h}(\zeta_a,\zeta_b)}{(\zeta_b-\zeta_a)}\chi \\ &\quad \left.\left. - \frac{g^2}{4(p\eta)}\frac{\tilde{H}(\zeta_a,\zeta_b)}{(\zeta_b-\zeta_a)}(\eta\theta)\chi\right\}\right|_{\theta=0} \end{aligned}$$

where we have used

$$\frac{\mathrm{d}\tau}{\mathrm{d}\zeta} = \frac{-1}{e(p\eta)}\left(1 + \mathrm{i}\frac{(\eta\theta)}{2e(p\eta)}\chi\right) \qquad \text{and} \qquad \chi\frac{\mathrm{d}\tau}{\mathrm{d}\zeta} = \frac{-\chi}{e(p\eta)} = \frac{\chi}{(\zeta_b - \zeta_a)} \tag{72}$$

and have defined the functions

$$\tilde{h}(\zeta_b, \zeta_a) = (A(\zeta_a) + A(\zeta_b))(\zeta_b - \zeta_a) - 2\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta\, A(\zeta) \tag{73}$$

and

$$\begin{aligned}\tilde{H}(\zeta_b, \zeta_a) = {}& \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta \frac{\left(\int_{\zeta_a}^{\zeta} \mathrm{d}\zeta\, A(\zeta)\right)^2}{(\zeta - \zeta_a)^2} - \frac{2\left(\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta\, A(\zeta)\right)^2}{(\zeta_b - \zeta_a)} - 2A(\zeta_a)\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta \frac{\left(\int_{\zeta_a}^{\zeta} \mathrm{d}\zeta\, A(\zeta)\right)}{(\zeta - \zeta_a)} \\ & + 3A^2(\zeta_a)(\zeta_b - \zeta_a) - \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta\, A^2(\zeta) - A(\zeta_a)A(\zeta_b)(\zeta_b - \zeta_a) + (A(\zeta_b) + A(\zeta_a)) \\ & \times \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta\, A^2(\zeta) + 2(\zeta_b - \zeta_a)\left[\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta \frac{A^2(\zeta)}{(\zeta - \zeta_a)} - \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta \frac{\left(\int_{\zeta_a}^{\zeta} \mathrm{d}\zeta\, A(\zeta)\right)^2}{(\zeta - \zeta_a)^3}\right].\end{aligned} \tag{74}$$

It is remarkable to note that

$$\lambda_b + \lambda_a = \eta\theta. \tag{75}$$

This condition reflects the fact that we maintain an antiperiodic boundary condition for the spin variables during the whole evolution. To be able to identify $\zeta = \eta x$ at the end points we use the integral representation $\delta\left(\zeta_b - \zeta_a + ep\eta - \mathrm{i}\frac{\eta\theta}{2}\chi\right) = \int \frac{\mathrm{d}p_{\zeta_b}}{2\pi} \exp\left(\mathrm{i}p_{\zeta_b}\left(\zeta_b - \zeta_a + ep\eta - \mathrm{i}\frac{\eta\theta}{2}\chi\right)\right)$ and next shift the momentum $p \longrightarrow p - \eta p_{\zeta_b}$. This gives the following delta function: $\delta(\zeta_b - \zeta_a - \eta(x_b - x_a))$, namely $\zeta_a = \eta x_a$ and $\zeta_b = \eta x_b$. Next, we integrate respectively over $\mathrm{d}\zeta_a$, $\mathrm{d}\zeta_b$, $\mathrm{d}\lambda_a$, $\mathrm{d}\lambda_b$, $\mathrm{d}e$ and $\mathrm{d}\chi$, the propagator is therefore reduced to

$$\begin{aligned}\tilde{S}(x_b, x_a) = {}& (2\mathrm{i}) \exp\left(\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right) \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p^2 - m^2 + \mathrm{i}\varepsilon} \left[\left(\int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta (\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a)\right.\right. \\ & \left. - \tilde{H}(\zeta_a, \zeta_b)\right) \frac{g^2}{4(p\eta)} \frac{(\eta\theta)}{(\zeta_b - \zeta_a)} + \frac{1}{2}(p\theta + m\theta^5) - \frac{\mathrm{i}g(A(\zeta_b) - A(\zeta_a))}{8(p\eta)(\zeta_b - \zeta_a)} \\ & \left. \times (\eta\theta)(\theta f \theta) - \frac{g}{4(p\eta)} \frac{\tilde{h}(\zeta_a, \zeta_b)}{(\zeta_b - \zeta_a)} (\theta f p)\right] \exp\left\{\mathrm{i}p(x_b - x_a) + \mathrm{i}\frac{g^2}{2p\eta}\right. \\ & \left.\left. \times \int_{\zeta_a}^{\zeta_b} \mathrm{d}\zeta (\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a) + \frac{g}{4(p\eta)}(A(\zeta_b) - A(\zeta_a))(\theta f \theta)\right\}\right|_{\theta=0}.\end{aligned} \tag{76}$$

We notice that the integration over the variable ζ is carried out along a straight line connecting ηx_a and ηx_b given by equation (53).

Finally, in order to perform differentiation with respect to θ^n, one should first expand the exponential as

$$\begin{aligned}\exp\left\{\frac{g}{4(p\eta)}(A(\zeta_b) - A(\zeta_a))(\theta f \theta)\right\} = {}& 1 + \frac{g}{4(p\eta)}(A(\zeta_b) - A(\zeta_a))(\theta f \theta) \\ & + 4\left(\frac{g}{4(p\eta)}\right)^2 ((A(\zeta_b) - A(\zeta_a)))^2 \left(f_{01}f_{23} - f_{02}f_{13} + f_{03}f_{12}\right) \theta^0\theta^1\theta^2\theta^3.\end{aligned} \tag{77}$$

Next, making use of the identity

$$\exp\left(\mathrm{i}\tilde{\gamma}^n \frac{\partial_l}{\partial\theta^n}\right) f(\theta)\bigg|_{\theta=0} = f\left(\frac{\partial}{\partial\beta^n}\right)\exp\left(\mathrm{i}\beta_n\tilde{\gamma}^n\right)\bigg|_{\beta=0} \tag{78}$$

where β_n are odd Grassmannian variables and f is an arbitrary function, we get the final expression for the causal propagator of a spinning particle in interaction with the plane wave field

$$\tilde{S}(x_b, x_a) = -\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\frac{\Phi_p(x_b, x_a)}{p^2 - m^2 + \mathrm{i}\varepsilon}\exp\left\{\mathrm{i}p(x_b - x_a) + \mathrm{i}\frac{g^2}{2p\eta}\int_{\zeta_a}^{\zeta_b}\mathrm{d}\zeta(\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a)\right\} \tag{79}$$

where $\Phi_p(x_b, x_a)$ is the so-called spin factor (Polyakov factor) given by

$$\begin{aligned}\Phi_p(x_b, x_a) = \Bigg\{&(p\tilde{\gamma} + m\gamma^5) + \frac{g^2}{2(p\eta)}\left[\frac{\int_{\zeta_a}^{\zeta_b}\mathrm{d}\zeta(\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a) - \tilde{H}(\zeta_a, \zeta_b)}{(\zeta_b - \zeta_a)}\right](\eta\tilde{\gamma})\\
&- \frac{g}{2(p\eta)}\frac{\tilde{h}(\zeta_a, \zeta_b)}{(\zeta_b - \zeta_a)}(\tilde{\gamma} f p)\\
&- \frac{g}{8(p\eta)}\left[\frac{\frac{g^2}{(p\eta)}\int_{\zeta_a}^{\zeta_b}\mathrm{d}\zeta(\zeta - \zeta_a)^2 K^2(\zeta, \zeta_a) - \tilde{H}(\zeta_a, \zeta_b) - 2\mathrm{i}}{(\zeta_b - \zeta_a)}\right]\\
&\times (A(\zeta_b) - A(\zeta_a))(\eta\tilde{\gamma})(\tilde{\gamma} f \tilde{\gamma}) + \frac{g}{4(p\eta)}(A(\zeta_b)\\
&- A(\zeta_a))[2(p f \tilde{\gamma}) - (p\tilde{\gamma})(\tilde{\gamma} f \tilde{\gamma}) - m\gamma^5(\tilde{\gamma} f \tilde{\gamma})]\\
&+ \frac{g^2}{8(p\eta)^2}\frac{\tilde{h}(\zeta_a, \zeta_b)(A(\zeta_b) - A(\zeta_a))}{(\zeta_b - \zeta_a)}[2(p\tilde{\gamma})(\eta\tilde{\gamma}) + (\tilde{\gamma} f p)(\tilde{\gamma} f \tilde{\gamma})]\\
&- \frac{mg^2}{4(p\eta)^2}((A(\zeta_b) - A(\zeta_a)))^2(f_{01}f_{23} - f_{02}f_{13} + f_{03}f_{12})\Bigg\}.\end{aligned} \tag{80}$$

It is easy to show that the function $\tilde{S}(x_b, x_a)$ and its corresponding causal Green function $S^c(x_b, x_a)$, namely $\tilde{S} = S^c\gamma^5$, verify respectively the corresponding Dirac equations:

$$(\gamma\pi_b - m)S^c(x_b, x_a) = -\delta^4(x_b - x_a) \tag{81}$$

where $\pi_\mu = (\mathrm{i}\partial_\mu - gA_\mu)$; g is the electronic charge and $[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$; $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$; $\mu, \nu = \overline{0, 3}$ or multiplying by γ^5 on both sides of (81), we get

$$\left(\tilde{\gamma}\pi_b - m\gamma^5\right)\tilde{S}(x_b, x_a) = \delta^4(x_b - x_a) \tag{82}$$

where $\tilde{\gamma}^\mu = \gamma^5\gamma^\mu$; $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \tilde{\gamma}^5$; $(\gamma^5)^2 = -1$.

The result given by expression (80) is, in principle, equivalent to that given in [9] where the so-called Polyakov spin factor $\Phi_p(x_b, x_a)$ is obtained by acting through the operator $(\hat{\Pi}_b + m)$ on the bosonic propagator solution of the quadratic Dirac equation.

3. Conclusion

In this paper we have been able to calculate, within the framework of path integrals, the exact and analytic Green function for a Dirac particle in interaction with the plane wave field expressed in the coordinate gauge. The problem has been enormously simplified using two identities; one bosonic and the other fermionic. These identities are directly related to the classical equations of motion which have naturally appeared as an argument of the delta functional. The calculations seem to be more difficult due to the complicated spinor structure describing the spinning degrees of freedom. But the difficulty is overcome by using exterior current sources. Then, we have been able to extract from the exact and analytic Green function the so-called Polyakov spin factor. Finally, let us note that it is interesting to extend our calculations to the case where the anomalous magnetic moment is present, a problem which is under consideration.

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